To solve these questions, we will use the concepts of geometric and binomial distributions, along with the normal approximation to the binomial distribution.

1. \*\*Probability that Sam’s first upgrade will occur after the third flight:\*\*

Since each flight represents a Bernoulli trial where Sam is either upgraded (success) or not upgraded (failure), and the probability of being upgraded is 0.10, we can model the situation with a geometric distribution. Specifically, the probability that the first upgrade happens after the third flight is the same as the probability that there are no upgrades in the first three flights.

Let \( p = 0.10 \), the probability of getting an upgrade. Then, the probability of not getting an upgrade on a single flight is \( 1 - p = 0.90 \).

The probability that Sam's first upgrade happens after his third flight is:

\[

P(\text{first upgrade after 3 flights}) = (1-p)^3 = 0.90^3 = 0.729

\]

2. \*\*Probability that Sam will be upgraded exactly 2 times in his next 20 flights:\*\*

Here, we use the binomial distribution, where the number of flights is \( n = 20 \) and the probability of success (upgrade) on each flight is \( p = 0.10 \).

The probability mass function for a binomial distribution is given by:

\[

P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}

\]

For \( k = 2 \):

\[

P(X = 2) = \binom{20}{2} (0.10)^2 (0.90)^{18}

\]

First, calculate \( \binom{20}{2} = \frac{20 \times 19}{2} = 190 \).

\[

P(X = 2) = 190 \times 0.01 \times 0.150858 = 0.286752

\]

3. \*\*Would you be surprised if Sam receives more than 20 upgrades to first class during the year (104 flights)?\*\*

To assess whether this outcome would be surprising, we can calculate the expected number of upgrades and approximate the distribution of the number of upgrades using the normal approximation to the binomial distribution, given the large number of flights.

The expected number of upgrades in 104 flights is:

\[

\mu = np = 104 \times 0.10 = 10.4

\]

The standard deviation is:

\[

\sigma = \sqrt{np(1-p)} = \sqrt{104 \times 0.10 \times 0.90} \approx 3.06

\]

To find out how surprising more than 20 upgrades would be, we calculate the z-score for 20.5 (using continuity correction):

\[

z = \frac{20.5 - 10.4}{3.06} \approx 3.32

\]

A \( z \)-score of 3.32 corresponds to a very small tail probability in a standard normal distribution (approximately 0.00045), suggesting that receiving more than 20 upgrades is indeed a rare event.

Therefore, yes, it would be surprising if Sam receives more than 20 upgrades in 104 flights.